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Quotient-3 Cordial Labelling For Cycle Related Graphs

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Abstract: Let G be a graph of order p and size q. Let f: V (G) $\rightarrow Z_4 - \{0\}$ be a function. For each E (G) define f*: E (G) $\rightarrow Z_3$ by f*(uv) = $\left\lceil \frac{f(u)}{f(v)} \right\rceil$ (mod 3) where f (u) $\geq$ f (v). If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G. $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$. Here we proved that C_n and some cycle related graphs like $[C_n ; C_3]$, n=3,6,9,…, $(P_2 \cup mk_1) + N_2$, S $(C_n ; S_2)$, joint sum of C_n and two cycles C_n having a common vertex is quotient-3 cordial.

Key words: cycle, joint sum, subdivision, quotient-3 cordial graph.

1. Introduction

Here the graphs considered are finite, simple, undirected and non trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [2] for more information. The cordial labeling concept was first introduced by cahit [1]. The quotient-3 cordial labeling have been introduced by P. Sumathi, A. Mahalakshmi and A. Rathi found in [5-7]. They found some family of graphs are quotient-3 cordial. For notations and terminology we follow [8]. If G receives quotient-3 cordial labeling then G is called as quotient-3 cordial graph. The number of vertices having label i denotes $v_f(i)$ and the number of edges having label k denotes $e_f(k)$, $1 \leq i \leq 3$, $0 \leq k \leq 2$.

2. Preliminaries

Definition: 2.1 $[C_n ; C_3]$ graph is obtained by attaching the cycle C_3 with every vertex of C_n.

Definition: 2.2

$(P_2 \cup mk1) + N_2$ graph is obtained with the vertex set $V=\{z_1,z_2,x_1,x_2 \ldots X_m\} \cup \{y_1,y_2\}$ and the edge set $\{[(y_1z_1), (y_1z_2), (y_2z_1), (y_2z_2), (z_1z_2)] \cup [(y_1x_1) \cup (y_2x_i) : 1 \leq i \leq m]\}$.

Definition: 2.3

A graph $(C_n ; S_2)$ is obtained by attaching the star S_2 with each vertex of a cycle C_n through an edge.

Definition: 2.4 From G, a graph S(G) is obtained by subdividing every edge of G with a new vertex is called subdivision of G.

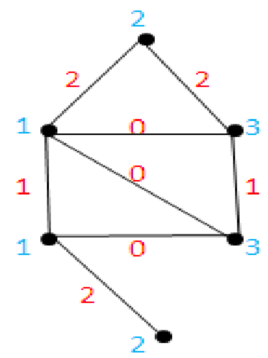
Definition: 2.5

A vertex of first copy of C_n is connected with a vertex of second copy of C_n through an edge is said to be joint sum of C_n.

3. Main Result

Definition: Let G be a graph of order p and size q. Let f: V (G) → Z_4 – {0} be a function. For each E (G) define f*: E (G) → Z_3 by f*(uv) = $\left\lceil \frac{f(u)}{f(v)} \right\rceil$ (mod 3) where f (u) ≥ f (v). If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G. $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$.

Illustration: 1 A quotient-3 cordial graph



Theorem: 3.1 All cycles C_n are quotient-3 cordial for $n \geq 4$ ($n \neq 9, 15, 21, \ldots$)

Proof: Let V(G) = { x_i : $1 \leq i \leq n$} and E(G) = {($x_n x_1$), ($x_i x_{i+1}$) : $1 \leq i \leq n-1$}.

Here $|V(G)| = |E(G)|$ = n.

Let f: V (G) → Z_4 - {0}

Case (i): when n ≡ 0, 1, 4, 5 (mod 6)

For all i,

$f(x_i) = 1$, $i \equiv 1, 2 \pmod 6$

$f(x_i) = 3$, $i \equiv 0, 3 \pmod 6$

$f(x_i) = 2$, $i \equiv 4, 5 \pmod 6$

Case (ii): when n ≡ 2 (mod 6)

Labeling of x_i, $1 \leq i \leq n-3$ are same as in case (i).

In this case, label the vertices x_{n-2}, x_{n-1}, x_n by 2, 1 and 3 respectively.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0 (mod 6)	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
n ≡ 1,4 (mod 6)	$\frac{n+2}{3}$	$\frac{n+2}{3}$ - 1	$\frac{n+2}{3}$ - 1
n ≡ 2,5 (mod 6)	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$ - 1

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 6$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod 6$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3}$	$\frac{n+2}{3} - 1$
$n \equiv 2 \pmod 6$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3} - 1$
$n \equiv 4 \pmod 6$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3}$
$n \equiv 5 \pmod 6$	$\frac{n+1}{3} - 1$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

Table 1

Theorem: 3.2 The graph $[C_n ; C_3]$ is quotient-3 cordial for n = 3, 6, 9, ….
Proof: Let $V(G) = \{[x_i: 1 \le i \le n] \cup [v_{ij}: 1 \le i \le n, j = 1, 2]\}$
$E(G) = \{[(x_1x_n), (x_ix_{i+1}): 1 \le i \le n-1] \cup [(x_iv_{ij}): 1 \le i \le n, j = 1, 2] \cup [(v_{i1}v_{i2}): 1 \le i \le n]\}$
Let $|V(G)| = 3n$, $|E(G)| = 4n$.
Define $f: V(G) \to Z_4 - \{0\}$
Labeling of x_i, $1 \le i \le n$ is given below.
$f(x_i) = 1$, $i \equiv 1, 2 \pmod 3$
$f(x_i) = 3$, $i \equiv 0 \pmod 3$
Labeling of v_{ij}, $1 \le i \le n$, $1 \le j \le 2$ is given below
$f(v_{ij}) = 1$, $i \equiv 1 \pmod 3$, $j = 1$
$f(v_{ij}) = 3$, $i \equiv 1 \pmod 3$, $j = 2$
$f(v_{ij}) = 3$, $i \equiv 0 \pmod 3$, $j = 1$
$f(v_{ij}) = 2$, $i \equiv 2 \pmod 3$, $j = 1, 2$
$f(v_{ij}) = 2$, $i \equiv 0 \pmod 3$, $j = 2$
For all n, $v_f(1) = v_f(2) = v_f(3) = n$
$e_f(0) = e_f(1) = e_f(2) = \frac{4n}{3}$.

Theorem: 3.3 A graph $(P_2 \cup nK_1) + N_2$ is quotient-3 cordial ($n \neq 2, 5, 8, \ldots$)
Proof: Let $V(G) = \{u_i, v_i, w_j: 1 \le j \le n, i = 1, 2\}$ and
$E(G) = \{[(u_1u_2), (v_1u_1), (v_1u_2), (v_2u_1), (v_2u_2)] \cup [(v_1w_j), (v_2w_j]: 1 \le j \le n]\}$
Let $|V(G)| = 4 + n$, $|E(G)| = 5 + 2n$.
Let $f: V(G) \to Z_4 - \{0\}$
$f(u_1) = f(v_1) = 1$
$f(u_2) = f(v_2) = 3$.
Label the vertices w_j, $1 \le j \le n$ as follows
$f(w_1) = 2$
When $n \equiv 0, 1 \pmod 3$
For $2 \le j \le n$,
$f(w_j) = 1$, $j \equiv 2 \pmod 3$
$f(w_j) = 2$, $j \equiv 0 \pmod 3$
$f(w_j) = 3$, $j \equiv 1 \pmod 3$

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Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 6$	$\frac{n+3}{3}+1$	$\frac{n+3}{3}$	$\frac{n+3}{3}$
$n \equiv 1 \pmod 6$	$\frac{n+2}{3}+1$	$\frac{n+2}{3}$	$\frac{n+2}{3}+1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 6$	$\frac{2n+3}{3}+1$	$\frac{2n+3}{3}$	$\frac{2n+3}{3}+1$
$n \equiv 1 \pmod 6$	$\frac{2n+1}{3}+1$	$\frac{2n+1}{3}+1$	$\frac{2n+1}{3}+1$

Table 2

Theorem: 3.4 $S(C_n;S_2)$ are quotient-3 cordial.

Proof: The cycle C_n has the vertices $u_1, u_2, u_3 \ldots u_n$. Let x_i be the vertices subdividing the edges (u_iu_{i+1}) for all $1 \le i \le n-1$ and the vertex x_n subdividing the edge u_nu_1.

Let $V(G) = \{[u_i, x_i: 1 \le i \le n] \cup [v_{ij}: 1 \le i \le n, j = 1, 2] \cup [w_{ij}: 1 \le i \le n, j = 1, 2]\}$ and $E(G) = \{[(u_nx_n), (x_nu_1), (u_ix_i), (x_iu_{i+1}): 1 \le i \le n-1] \cup [(u_{2i-1}v_{ij}): 1 \le i \le n, j = 1, 2] \cup [v_{ij}w_{ij}): 1 \le i \le n, j = 1, 2]\}$

Let $|V(G)| = |E(G)| = 6n$.

Define $f: V(G) \to Z_4 - \{0\}$ by

For all i, $1 \le i \le n$ and $j = 1, 2$

$f(u_i) = f(x_i) = 3$

$f(v_{ij}) = 1$

$f(w_{ij}) = 2$

For all n, $v_f(1) = v_f(2) = v_f(3) = 2n$

$e_f(0) = e_f(1) = e_f(2) = 2n$.

Theorem: 3.5 Joint sum of C_n are quotient-3 cordial.

Proof: The first cycle C_n has the vertices $u_1, u_2, \ldots, u_n$ and an another cycle C_n has the vertices $u_{n+1}, u_{n+2}, \ldots, u_{2n}$.

Let $V(G) = \{[u_i: 1 \le i \le 2n]\}$.

$E(G) = \{[(u_1u_n), (u_iu_{i+1}): 1 \le i \le n-1] \cup [(u_1u_{n+1})] \cup [(u_iu_{i+1}), (u_{n+1}u_{2n}): n+1 \le i \le 2n-1]\}$

Let $|V(G)| = 2n$ and $|E(G)| = 1 + 2n$

Let $f: V(G) \to Z_4 - \{0\}$

Case (i): when $n \equiv 0, 2, 3, 5 \pmod 6$

For $1 \le i \le 2n$

$f(u_i) = 1, \quad i \equiv 1, 2 \pmod 6$

$f(u_i) = 3, \quad i \equiv 0, 3 \pmod 6$

$f(u_i) = 2, \quad i \equiv 4, 5 \pmod 6$

Case (ii): when $n \equiv 1 \pmod 6$

For $1 \le i \le n$

$f(u_i) = 2, \quad i \equiv 1, 2 \pmod 6$

$f(u_i) = 3, \quad i \equiv 0, 3 \pmod 6$

$f(u_i) = 1, \quad i \equiv 4, 5 \pmod 6$

For $n+1 \le i \le 2n$

$f(u_i) = 3$, $i \equiv 1, 4 \pmod 6$
$f(u_i) = 1$, $i \equiv 2, 3 \pmod 6$
$f(u_i) = 2$, $i \equiv 0, 5 \pmod 6$
In this case after labeling all the vertices interchange the label of u_{n+3} and u_{n+4}.

Case (iii): when $n \equiv 4 \pmod 6$
Labeling of the vertices u_i, $1 \le i \le 2n-1$ $[i \neq n]$ are same as in Case (i)
Here $f(u_n) = 3$, $f(u_{2n}) = 2$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0,3 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 6$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
$n \equiv 2 \pmod 6$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
$n \equiv 4 \pmod 6$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$
$n \equiv 5 \pmod 6$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3}+1$	$\frac{2n}{3}$
$n \equiv 1,4 \pmod 6$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
$n \equiv 2 \pmod 6$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$	$(\frac{2n+2}{3})$
$n \equiv 3 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}+1$
$n \equiv 5 \pmod 6$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$

Table 3

Theorem: 3.6 Two copies of cycle C_n having a common vertex is quotient-3 cordial.
Proof: Let $V(G) = \{[u_i: 1 \le i \le 2n-1]\}$ and
$E(G) = \{[(u_1u_n), (u_iu_{i+1}): 1 \le i \le n-1] \cup [(u_1u_{n+1}), (u_1u_{2n-1}), (u_iu_{i+1}): n+1 \le i \le 2n-2]\}$
Let $|V(G)| = 2n-1$ and $|E(G)| = 2n$
Let $f: V(G) \rightarrow Z_4 - \{0\}$
Case (i): When $n \equiv 0 \pmod 6$
$f(u_{2n-1}) = 3$.
For $1 \le i \le 2n-2$
$f(u_i) = 1$, $i \equiv 1, 2 \pmod 6$
$f(u_i) = 3$, $i \equiv 0, 3 \pmod 6$
$f(u_i) = 2$, $i \equiv 4, 5 \pmod 6$
Case (ii): when $n \equiv 1 \pmod 6$
$f(u_{2n-3}) = 2$
$f(u_{2n-2}) = 1$
$f(u_{2n-1}) = 3$
For $1 \le i \le 2n-4$
$f(u_i) = 1$, $i \equiv 0, 1 \pmod 6$
$f(u_i) = 3$, $i \equiv 2, 5 \pmod 6$

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$f(u_i) = 2$, $i \equiv 3, 4 \pmod 6$
Case (iii): when $n \equiv 2 \pmod 6$
$f(u_{2n-2}) = 2$.
Labeling of the vertices u_i, $1 \le i \le 2n-1 (i \ne 2n - 2)$ are same as in case (i).

Case (iv): when $n \equiv 3 \pmod 6$
$f(u_{2n-1}) = 2$.
Labeling of the vertices u_i, $1 \le i \le 2n-2$ are same as in case (i).
Case (v): when $n \equiv 4 \pmod 6$
Labeling of the vertices u_i, $1 \le i \le n-1$ are same as in case (i) and $f(u_n) = 3$.
Labeling of the vertices u_i, $n+1 \le i \le 2n-1$ is given below.
For $1 \le i \le 2n-1$
$f(u_i) = 2$, $i \equiv 1, 2 \pmod 6$
$f(u_i) = 3$, $i \equiv 0, 3 \pmod 6$
$f(u_i) = 1$, $i \equiv 4, 5 \pmod 6$
Case (vi): when $n \equiv 5 \pmod 6$
$f(u_{2n-3}) = 3$, $f(u_{2n-2}) = 1$, $f(u_{2n-1}) = 2$.
Labeling of the vertices u_i, $1 \le i \le 2n-4$ are same as in case (i).

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3} - 1$	$\frac{2n}{3}$
$n \equiv 1 \pmod 6$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 2,5 \pmod 6$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
$n \equiv 3 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3} - 1$
$n \equiv 4 \pmod 6$	$(\frac{2n-2}{3})$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0,3 \pmod 6$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 6$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 2 \pmod 6$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$
$n \equiv 4 \pmod 6$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 5 \pmod 6$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$

Table 4

4. Conclusion

In this paper, a quotient-3 cordial for some standard graphs has been found. The quotient-3 cordial labeling of some more graphs and graph families shall be explored in future.

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