A review: Analysis of fractional order chaotic systems

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Abstract—**Chaos is generally understood as confusion or disorder. It is the property of systems which are complex andunpredictable in nature.Chaos is often misunderstood as noise, but they are different entities due to the predictable nature of chaos. In this paper different fractional order chaotic systems Fractional Calculus is not a new topic though but the use of it has been applied efficiently in the last few decades. Generally, real objects are fractional order systems, thus fractional order has been implemented on the different chaotic systems which are discussed in the paper and the changes on the systems are studied accordingly. The chaotic system's behavior is observed for fractional order. Each system's chaotic nature is studied for different fractional orders and analyzed to check which order is suitable to observe chaos in the respective chaotic system. Synchronization and applications of the fractional order systems are also discussed.**

I. INTRODUCTION

Chaos was first introduced in 1963 by E.Lorenz[8] Since then chaos has been the center of research for many scientists and research groups around the globe but it has been just a few decades that it's been used efficiently especially in the field of communication. Chaos has played an important role in control applications such as controlling of irregular behavior in devices and systems [5] even in communication privacy with the principle of chaotic masking [17], and in synchronization of two same or different systems which leads to secure communication, chaotic band radio and encryption [5]. Chaotic systems also have applications in short term predictions which can be implemented for predicting the weather, contagious diseases & economy

A Chaotic system comprises of differential equation where any two solutions at a time are way different for two very similar conditions. Many chaotic systems have been designed to generate chaos for purposes as discussed above. These chaotic systems generate attractors which are nothing but a set of points in phase space, invariant under the dynamics. These attractors are non-periodic in nature hence the conservative systems can't produce them [20].In nonautonomous system chaos may occur for even when there are just 2 variables in the system and in the case of autonomous it can happen with

even just three state variables [1]. Most of the times the chaotic systems are of integer order but by studying fractional order calculus it was observed that fractional order chaotic systems are better in the realization of the real objects due to their dynamic nature. The fractional order chaotic systems were found to be more adequate for describing the non-linearsystems in nature as compared to integer order chaotic systems. The fractional order systems are nonlocal in nature since their future state doesn't depend only on their present state but also on the conditions of its past states. [24]

Fractional order calculus is not a new topic. It is almost as old as integer order calculus. In the year 1695, Leibnitz wrote in a reply to de l'Hospital "Thus it follows that $d^{1/2}x$will be equal to $\sqrt{dx: x, \dots}$ from which one-day useful consequences will be drawn". We are familiar with integer order differential and integral operations but what if the order is a non-integer like a fraction or a constant like π thus fractional order calculus came into play. There are numerous methods to study fractional calculus, but the best-known method is the Riemann-Liouville which is given by

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_{\alpha}^{t}\frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau, \qquad (1.1)$$

for n-1 $\leq \alpha <$ n, where $\Gamma(.)$ is the gamma function [6] [21]. The main idea is to generalize the differentiation and integration to a fractional(non-integer) order which can be defined by [6]

$$aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}}, & R(\alpha) > 0 \\ 1, & R(\alpha) > 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha}, & R(\alpha) > 0 \end{cases} \qquad (1.2)$$

here $aD_t^{\alpha}$is the fundamental operator with a and t as limits, α is the order for operation where $\alpha \in$ R.

The functions used in the fractional calculus are Beta function, Gamma function and Mittag-Leffler Function. [13]

The basic form of the Gamma function is nothing but the rationalization of the fractional order calculus for all the existing real numbers. The Gamma function can be understood with the help of the following equation. [13] for all z Є R

$$\Gamma(z) = \int_0^\infty e^{-u} u^{(z-1)} du \qquad (1.3)$$

Beta Function can also be referred to be as Euler integral of the first kind. Beta function's solution can be defined with the Gamma function's help. The following equation represents the Beta integral and its solution in the terms of Gamma Function. [10]

$$B(r,s) = \int_0^1 (1-v)^{r-1} v^{s-1} du = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)} = B(s,r) (1.4)$$

where $r, s \in R$.

Mittag-Leffler Function is utilized prominently in the field of fractional calculus. An analogous role is played by this function in fractional calculus. The equation consists of an exponential function which has its own unique form. The Mittag-Leffler Function's standard equation is as follows [10]

$$E_\alpha(z) = \frac{\sum_{k=0}^\infty z^k}{\Gamma(\alpha k + 1)} (1.5)$$

Even though the fractional order chaotic systems are more dynamic in nature as compared to integer order they can't be directly implied in time-domain simulations. Thus, various methods were used introduced and are available in the literature, but the Laplace transformation of the Riemann Liouville method is the simplest of all which can be written as

$$\int_0^\infty e^{-st} {}_0 D_t^\alpha = s^\alpha L\{f(t)\} - \sum_{k=0}^{n-1} (s_0^k) D^{\alpha-k-1} f(t)|_{t=0} (1.6)$$

for n-1 ≤ α < n, where s=jω denotes the Laplace variable. If the initial conditions are set to zero, then the formula reduces to

$$L\left\{\frac{d^\alpha f(t)}{dt^\alpha}\right\} = s^\alpha L\{f(t)\} \qquad (1.7)$$

Where α is the fractional order and can be portrayed in the frequency domain by the transfer function $F = \frac{1}{s^\alpha}$. [2] Still being the simplest method it can't be directly implemented on electronic circuits thus an effective approach to overcome this issue is by approximating fractional operators by using standard integer order operators. Another method is the

approximations of the $1/s^\alpha$ where α Є (0,1) were given in the [09] with errors of approximately 2 dB for example for the order α=0.9 the transfer function will be [9]

$$\frac{1}{s^{0.9}} = \frac{2.2675(s+215.4)(s+1.292)}{(s+2.154)(s+0.01292)(s+359.4)} \qquad (1.8)$$

Similarly, linear transfer function approximations were done for fractional order α with maximum discrepancy 3dB thus one way to study fractional order chaotic systems isvia linear approximations.

II. FRACTIONAL ORDER CHAOTIC SYSTEMS

A. Fractional Lorenz System

After Lorenz introduced chaos he designed a chaotic system in 1963.The fractional order chaotic Lorenz system will be having the following equations.

$$\frac{d^q x}{dt^q} = \sigma(y - x)$$
$$\frac{d^q y}{dt^q} = rx - xz - y \qquad (2.1)$$
$$\frac{d^q z}{dt^q} = -bz + xy$$

here 'q' is taken as fractional order and q Є (0,1). In the case of Lorenz chaotic system, it was observed that at q=0.99 the attractors obtained were better than that of integer order and 0.91 was the fractional order threshold below which suitable chaotic behavior was not obtained thus to obtain best possible attractors via fractional order Lorenz chaotic system the dimension order should be 0.99*3=2.97 (for x, y and z dimensions combined). The effective dimension of ∑=2.91 is the threshold of the Lorenz system but it can't be termed as universal threshold for all the systems as each system will behave differently to fractional order [30]. Thus, it was found out that the effective dimension can be a non-integer value and the dynamics of the system depend on it and is highly sensitive to the fractional order. [41]
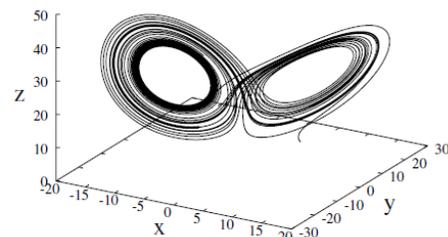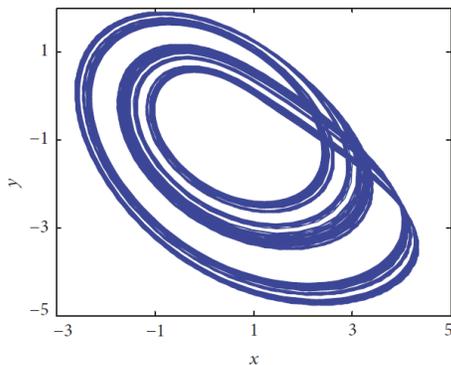

Fig. 2.1 Fractional order Lorenz Chaotic attractor

B. Fractional order Rossler System

Fractional order Rossler system was researched and it was analyzed numerically using the ADM (Adomian

Decomposition) method and the further realized on DSP board [31]. ADM method has been used on other chaotic systems (such as Lorenz, Chen, etc.) as it can deal with both linear and non-linear systems in time domain. ADM method was used as it had better accuracy, faster convergence and used far less computer resources hence ADM was used to find and answer to the fractional order chaotic systems rather than Adams-Bashforth-Moulton predictor corrector approach. When compared with integer order chaotic systems (q=1) the system has unique chaotic behavior and dynamic characteristics such as tangent bifurcations and period-doubling. It was observed that the fractional order threshold for chaotic behavior of the Rossler system was of $\sum$▦=1.119 (dimension of the system) and q=0.346 when a=0.55, b=2 and c=4 thus for q ∈ [0.346,1] the system remains in state of chaos. [31]

Fig 2.2 Chaotic attractor of Fractional order Rossler system

Following are the differential equations of the system

$$\frac{d^q x}{dt^q} = -y - z$$

$$\frac{d^q y}{dt^q} = x + ay \qquad (2.2)$$

$$\frac{d^q z}{dt^q} = b + z(x - c)$$

C. Fractional order Chen System

Chen chaotic system was studied for fractional order [23] by researchers due to the interesting finding which showed that Chen chaotic system showed chaotic behavior even for q=0.3 which is the least order for which chaotic nature is observed for any system yet. Laplace Transform approach was used to implement fractional order on Chen chaotic system in [33] where the $F = \frac{1}{s^q}$ [3-4] transfer function is used in frequency domain as fractional order integrals and differentials don't allow direct implementation in time domain as discussed in [2]. The

fractional order chaotic system can be defined with the following differential equations [35]

$$\frac{d^q x}{dt^q} = a(y - x)$$

$$\frac{d^q y}{dt^q} = cy - xz + (c - a) \qquad (2.3)$$
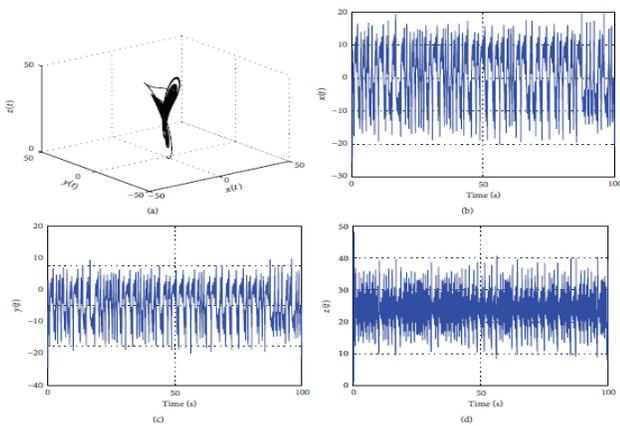
$$\frac{d^q z}{dt^q} = -bz + xy$$
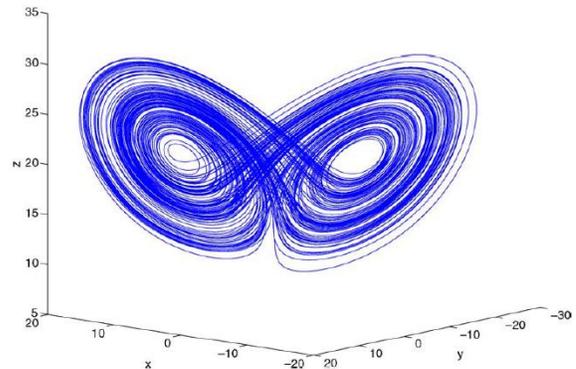
Fig.2.3 Chaotic attractor for Fractional order Chen system at q =0.8

D. Fractional order Unified chaotic system

Fractional order circuit implementation of unified chaotic system was studied in [4] at q=0.9($\sum$=2.9) and for v=0.5,0.8 and 1 the Lyapunov exponents were 0.0214,0.0302 and 0.0223 respectively. Even here the Laplace transform approach was used to design the fractional order system. Using circuit theory in Laplace domain the tree shape model values for R&C were obtained. Hence by replacing the capacitor with this model in the respective integer order system circuit the fractional order implementation was done in [4]. The threshold fractional order to obtain chaos was observed as 2.76 [38] with the fractional order q1 =0.93, q2=0.94 and q3=0.95 for x, y and z dimension respectively. The differential equations will be as follows

$$\frac{d^{q1} x}{dt^q} = (25\gamma + 10)(y - x)$$

$$\frac{d^{q2} x}{dt^q} = (28 - 35\gamma)x - xz + (29\gamma - 1)y \quad (2.4)$$

$$\frac{d^{q3} z}{dt^q} = xy - (\gamma + 8)z/3$$

**Fig.2.4** Chaotic attractor and time series output of Fractional order Unified chaotic system for γ=0.5

Chaos is observed in any system when the Lyapunov exponents are greater than zero.

E. *Fractional order Lü Chaotic system.*

The fractional order differential equations of the Lü system are [36]

$$\frac{d^q x}{dt^q} = a(y - x)$$

$$\frac{d^q y}{dt^q} = cy - xz \qquad (2.5)$$

$$\frac{d^q z}{dt^q} = -bz + xy$$

here the fractional order is q and (a,b,c) are parameters of the system whose values are (36,3,20) and predefined conditions are x= 0.2, y = 0.5 and z = 0.3.
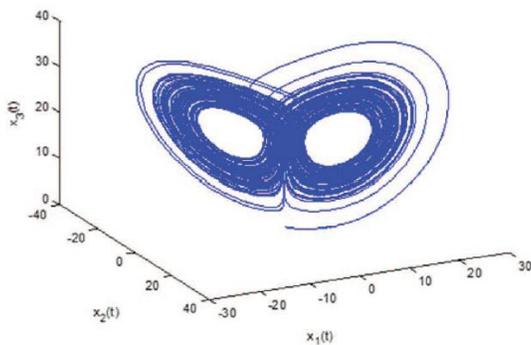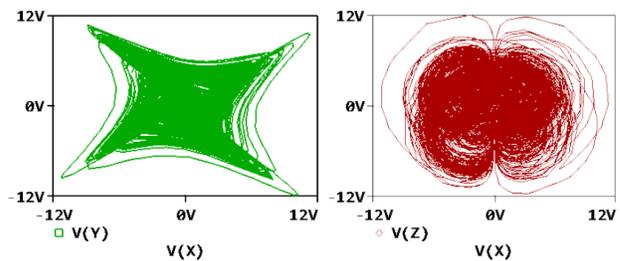

Fig.2.5 Chaotic attractor for Fractional order Lü system for q = 0.96

F. *Fractional order Pehlivan Chaotic system*

The Pehlivan system was implemented for fractional order in [2] which used the Laplace transform approach as used earlier in unified chaotic system. Here fractional order q=0.9 (2.7 order system) was the fractional order used and attractors, phase portraits, time series and FFT spectrum are compared with its integer order counterpart. The tree shape model was used to replace the capacitors in Pehlivan integer order chaotic circuit and the values of R&C were obtained using circuit theory in Laplace domain where it was seen that the fractional order system had better bandwidth response as compared to integer order and even the attractors were denser than that or integer order chaotic system. The differential equations of this Pehlivan system are

$$\frac{d^q x}{dt^q} = ax + by + yz$$

$$\frac{d^q y}{dt^q} = -ay - bx + xz \qquad (2.6)$$

$$\frac{d^q z}{dt^q} = z - xy$$

Fig. 2.6 X-Y and X-Z plane chaotic attractors of fractional order Pehlivan system.

G. *Fractional order Chua Chaotic system.*

The fractional order differential equations of the Chua Chaotic systems are [34]

$$\frac{d^q x}{dt^q} = a(y + \frac{x - 2x^3}{7})$$

$$\frac{d^q y}{dt^q} = x - y + z \qquad (2.7)$$

$$\frac{d^q z}{dt^q} = -by$$

The approximations were used from Table 2 for the fractional order and the Laplace transform approach was followed [2-4]. The value of b is found out to be 100/7. The double scroll attractor was visualized as follows
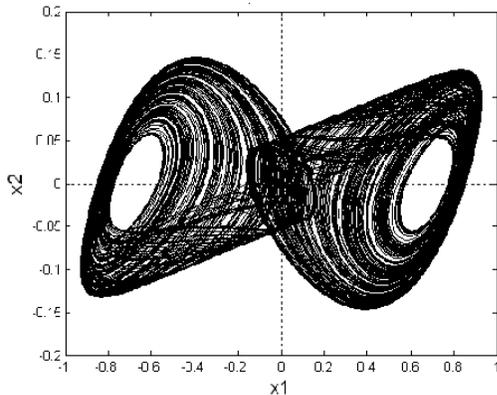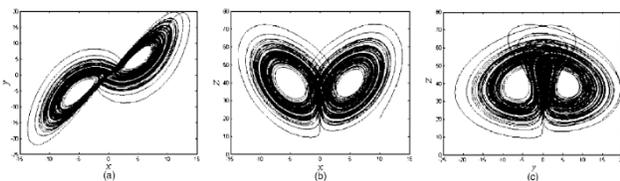
Fig.2.7 Chaotic attractor for Fractional order Chua system.

H. Fractional order Liu Chaotic System

The fractional order Liu system can be represented as follows

$$\frac{d^{q1}x}{dt^q} = a(y - x)$$

$$\frac{d^{q2}x}{dt^q} = bx - lxz \qquad (2.8)$$

$$\frac{d^{q3}z}{dt^q} = cz + hx^2$$

where q1, q2 and q3 are fractional orders of x, y and z parameters respectively and has (0,0,0) as an equilibria point. [39]

Fig 2.8 Chaotic attractor for Fractional order Liu system at q1 = q2 = q3 = 0.85

III Synchronization of the Chaotic systems.

Synchronization means when a thing occurs simultaneously or operates in unison [23]. It is a major application of fractional order chaotic system. There are 4 generations of chaotic secure communication systems, the first three generations are based on continuous chaotic synchronization and the fourth generation is based on impulsive methods to improve the bandwidth efficiency than the earlier generations [16]. The first generation is known as chaotic shift keying or additive chaos masking the second generation is known as chaotic modulation the third is called as chaotic cryptosystem. Thus, it was found out that continuous synchronization is less robust than impulsivesynchronization.

Hence, synchronization of the two same fractional order systems or two different fractional order systems or between a fractional order and an integer order system is possible using various synchronization techniques. [43-46]. Synchronization of fractional order Lorenz chaotic system is studied in [41] using master slave method. Synchronization of the fractional order Lorenz and Chen chaotic system is studied and discussed in [44]. The fractional order Chen and Lü system were synchronized by the fractional order sliding mode control method in [23]. The synchronization of fractional order Rossler system was studied in [46] and it was used convert the chaos signal into required signal which is bounded and is also a useful signal. Synchronization of fractional order Liu chaotic systems is carried out in [40] using the adaptive control method and in [39] using linear coupling method. In [34] fractional order Qi and Lü are non-linearly synchronized and numerical methods were performed to monitor the efficiency of the used synchronized method. Unified chaotic system's synchronization was studied in [44] using impulsive control method and later mathematical simulations are carried out to test the feasibility of the synchronized system. In [43] the flexibility of synchronization was tested as the user could switch between fractional order and integer order depending upon on the desired goal to be achieved. In [45] a new fractional order chaotic system was tested for synchronization. This was Lorenz-Stenflo (LS) system, then using the stability theory the synchronization was carried out with mathematical calculations and simulations for verification.

IV Applications of chaotic systems

The major application of chaotic systems are stabilization and control using the extreme sensitivities of the systems as the output of the chaotic systems heavily depend on the changes made in the initial conditions [48]. Synchronization of 2 chaotic signals can be used for encryption by superimposing a message on one of the signals, then on the receiving end only the user with other sequence can decode the message by removing the chaotic masking component.

Chaotic oscillators and controllers can be designed with the help of fractional order chaotic systems [37]. Fractional order systems can be also used for spreading the signal which will in turn cause higher security of the message signal. A signal can be hidden in the noise and be transmitted at low power which increase message privacy between the sender and receiver.

Chaotic systems are used for short term prediction and hence play a major role in predicting diseases, economy, weather, etc. [48]

Neural networks learn by their own just like biological systems hence this approach can be applied for prediction and control such as researchers have already started the enhancement of handwriting recognition using chaotic neural networks.

Fuzzy logic can also be implemented on various fractional order chaotic systems in the control domain.

Random number generators (RNG) [52] can be designed with the fractional order chaotic systems [22] and using the RNG based encryption algorithms chaos can be implemented in various field such as image processing [50], physics and mechatronics. Basically, fractional order system has similar applications as the integer order chaotic systems, but they are efficient than the integer order systems due to their highly dynamic nature which mimics the real objects or systems in the nature thus, giving the user the choice to choose between them both depending upon the desired purpose. Hence, with the help of chaos we can design systems that are as flexible as the biological systems in the nature, but the advantages should more than enough to counter the complexity introduced due to incorporation of the fractional order systems.

CONCLUSION

In this review, we have studied and discussed various fractional order chaotic systems. We have studied different chaotic systems with the help of their differential equations discussed how the incorporation of fractional order changes the chaotic behavior of the already highly sensitive non-linear systems. It was seen that the each chaotic has its own unique chaotic attractor. A small change in the system cause a considerable change in the output of the system.

With this analysis, we can conclude that fractional order improves the dynamic nature of the already sensitive chaotic systems. Different techniques of fractional order implementations were studied and discussed and with the help of these methods oth-er chaotic systems can also be realized and implemented with fractional order.

FUTURE SCOPE

There are various other chaotic systems which can be implemented with fractional order and can be studied and discussed thus can be incorporated in such reviews.

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