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Unique Enresdowed Graphs

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Abstract

Let G = (V, E) be a non empty, finite, simple graph. k - γ_r enresdowed graph is one in which every restrained dominating set of cardinality k contains a minimum restrained dominating set. In this paper, we discovered a few outcomes for the k - γ_r enresdowed graph related to private neighborhood.

Keywords : Dominating set , Domination number , Restrained Domination , Private Neighbour , enresdowed graphs.

1. Introduction

Let G = (V, E) be a non empty, finite, simple graph. A subset D of V(G) is called a dominating set of G if for every v ∈ V – D, there exists u ∈ D such that u and v are adjacent [3]. The minimum cardinality of the dominating set is called the domination number and it is denoted by γ(G). The Restrained dominating set of a graph is a dominating set in which every vertex in V – D is adjacent to some other vertex in V – D. The minimum cardinality of the restrained dominating set is called the restrained domination number and it is denoted by γ_r(G).The minimum restrained dominating set is called as γ_r set[7]. An element u of pn(x,X) for x in X is called a private neighbor of x relative to X and is one either u is an isolate of G[X] in which case u = x or u ∈ V – x and is adjacent to precisely one vertex of X. The open neighborhood N(v) of a vertex v in a graph G is the set of all vertices adjacent to v in G. The closed neighborhood N[v] of v is the set N(v) ∪{v}.

A restrained dominating set of a graph G containing a γ_r set of G is called a γ_r – enresdowed restrained dominating set of G. If that set is of cardinality k then it is called a k - γ_r enresdowed restrained dominating set[8]. A subset S of the vertex set in a graph G is said to be independent if no two vertices in S are adjacent[5]. The maximum number of vertices in an independent set of a graph G is called the independence number of G and it is denoted by $\beta_0(G)$.

Definition 1.1

Let k be a positive integer . A simple graph G = (V, E) is called a k - γ_r enresdowed graph if every restrained dominating set of G of cardinality k contains a γ_r set of G [8].

2. Main Results

Remark 2.1

1. Any graph of order p is p - γ_r enresdowed and γ_r - γ_r enresdowed.
2. If G is k - γ_r enresdowed then , $k \geq \gamma_r(G)$.
3. If $\gamma_r(G) \leq \Gamma_r(G)$, then G is $\Gamma_r - \gamma_r$ enresdowed .
4. Let P_p be a path on p vertices , then P_p is k - γ_r enresdowed where

$$k = \begin{cases} n+2 & \text{if } p = 3n \\ n+1 & \text{Otherwise} \end{cases}$$

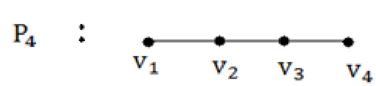
Remark 2.2

Let G be a k - γ_r enresdowed graph , and β_0 is the maximum number of vertices in an independent set. β_0 and k are not related.

Illustration 2.3

Three different situations (i) $k = \beta_0$ (ii) $k > \beta_0$ (iii) $k < \beta_0$ are given in the following examples.

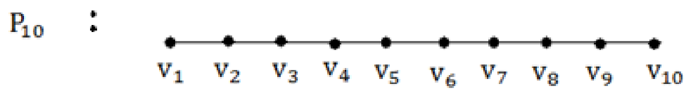
(i) In P_4 , γ_r- set is the set of pendant vertices such that $\gamma_r(P_4) = \beta_0(P_4)$.



Let D = {v_1 , v_4}, clearly D is a γ_r- set and β_0 - set.
$\therefore \gamma_r = \beta_0 = 2$. Here $k = 2$.
Therefore $k = \beta_0$ for P_4.

(ii) In P_6 , $\gamma_r(P_6)$ is the set of two pendant vertices and two leaf vertices so that $\gamma_r(P_6) = 4$ and where as in the case of β_0 the alternate vertices has been choosed .Thus $\beta_0(P_6) = 3$, also $k = 4$. Therefore $k > \beta_0$ for P_6 .

(iii) In P_{10} , D = {v_1, v_4, v_7, v_{10}} is a γ_r set where $k = 4$, and D_1 = {v_1, v_3, v_5, v_7, v_9} is a β_0 set . Hence $\beta_0 = 5$. Therefore $k < \beta_0$ for P_{10}.



3.Theorems on k - γ_r enresdowedness of graphs with its private neighbors

Theorem 3.1

Let G have no isolates , suppose G has a unique minimum restrained dominating set and s pendant vertices then G is not k - γ_r enresdowed for any k ,

$$p - s \leq k \leq p - 1$$

Proof

Let D be the unique γ_r set of G. Then there exist the following cases

Case (1)

If $p - s < \gamma_r$, then it is not $(p - s)$ - γ_r enresdowed. If $p - s > \gamma_r$, then S ∪ K will form a restrained dominating set which will not contain the unique dominating set D.

Let S = { pendants of G}.

If suppose $|S| = 1$, then $p - s = p - 1$. Then , it is not $(p - s)$ - γ_r enresdowed.

If $|S| = s$, $s \geq 2$.

Let $K' = \{v \mid d(v) \neq 1$ and v is not adjacent to pendants$\}$.

Choose $K \subseteq K'$ such that

$|K| = p - 2s$. Here $S \cap K = \varphi$.

Then $|S \cup K| = |S| + |K| - |S \cap K|$

$= s + p - 2s$

$|S \cup K| = p - s$.

is the restrained dominating set. But it will not contain the point adjacent to the pendant vertex, which lies in the γ_r set. Thus in this case, G is not $(p - s)$ - γ_r enresdowed. i.e G is not k - γ_r enresdowed for $k < p - s$.

Case (2)

If $\gamma_r < p - 1$,

Let S = {pendant vertices of G}.

Let $K' = \{v \mid d(v) \neq 1$ and v is not adjacent to pendants$\}$.

If $|S| = l$ and $|\gamma_r| = k$, then G is not k - γ_r enresdowed for any k,

$\gamma_r + j \leq k \leq p - 2$, with $j = 1,2,3,\ldots\ldots , p - (l + 3)$.

Definition 3.2

Consider a path of k vertices namely, $u_1, u_2, \ldots\ldots, u_i \ldots\ldots, u_k$ to the vertex u_i , $1 \leq i \leq k$, attach a fan f_{1,s_i} , $s_i \geq 4$. The graph obtained is called n - fan graph and it is

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denoted by $D_{s_1,s_2,......,s_i......,s_k}$.The vertices $u_1, u_2,......, u_i......,u_k$ are called nodes of the n - fan graph.

Remark 3.3

The n - fan graph has a unique γ_r set.

Definition 3.4

Let S be a set of vertices . Let u ∈ S , a vertex v ∈ V(G) is said to be a private neighbor of u with respect to S if N[v] ∩ S = {u}. Furthermore, the private neighbor set of u with respect to S is denoted by pn(u,S) = {v : N[v] ∩ S = {u}}[7].

Notation 3.5

Let G be a n - fan graph and D = $\{u_1, u_2,......, u_i ,......,u_k\}$ be a unique γ_r set of G. Let $\eta = \min_{1\le i \le k} \{|pn(u_i , D)|\}$, where $pn(u_i , D)$ is the private neighbour set of u_i with respect to D.

Theorem 3.6

Let G be a n - fan graph with nodes $u_1, u_2,......, u_i ,......,u_k$, then every node has atleast four private neighbors in V – D.

Proof

Let D be the set of nodes. Therefore D = $\{u_1, u_2,......, u_i ,......,u_k\}$. D is a unique γ_r set of G, for every u_i ∈ D and for any $v_j \in pn(u_i , D)$ is adjacent with $v_{j+1} \in pn(u_i , D)$.

Suppose u_i has no private neighbor in V – D with respect to D. Since u_i is not an isolate or pendant of G, there exists a vertex $v_j \in N(u_i) \cap (V - D)$ which is adjacent to any vertex of the set $\{u_1, u_2, ..., u_{i-1} , u_{i+1} ......, u_{k-1}\}$. Then there exist a set $D_1 = \{u_1, u_2,......, u_{i-1} , v_j , u_{i+1} ......,u_k\}$. u_i is dominated by v_j and any neighbor of u_i is dominated by D_1 - v_j . Therefore D_1 is a γ_r set of G , a contradiction . Therefore every u_i ∈ D has a private neighbor in V – D. with respect to D.

Suppose if u_i has exactly one private neighbor v_j in V – D with respect to D. Then v_j must be either a pendant or it is adjacent to some vertex v_{j+1} in V – D , where v_{j+1} is adjacent to any u_j ∈ D , 1≤ j ≤ k -1 . Then the graph G is not unique .

Suppose if u_i has exactly two private neighbors v_j and v_{j+1} in V – D with respect to D. Then the vertices u_i, v_j and v_{j+1} form a cycle C_3. Then the graph G is not unique.

Suppose u_i has exactly three private neighbors in V – D with respect to D. Let $\{v_{j-1} , v_j , v_{j+1}\}$ be the set of three adjacent private neighbors of u_i, then there exist a set

$D_2 = \{u_1, u_2, \ldots\ldots, u_{i-1}, v_j, u_{i+1} \ldots\ldots, u_k\}$. Choose v_j such that $d(v_j) = \Delta$, then v_j is adjacent with $u_i \in D$ and also with v_{j-1} and v_{j+1} in $V - D$. v_j dominates u_i and its neighbors v_{j-1} and v_{j+1} and v_{j-1}, u_i, v_{j+1} are adjacent in $V - D$. Therefore D_2 is a γ_r set of G, a contradiction. Therefore every $u_i \in D$ has at least four private neighbors in $V - D$ with respect to D.

Theorem 3.7

Let G be a n - fan graph with nodes $u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_k$. Let $\eta = \min_{1\le i \le k} \{|pn(u_i, D)|\}$, then there exist an s, $1 \le s \le k$ such that $pn(u_s, D) = \eta$. Then $(D - \{u_s\}) \cup (pn(u_s, D) - \{u_{sj}\})$, $u_{sj} \in pn(u_s, D)$ is a restrained dominating set of G of cardinality $k + \eta - 2$.

Proof

Let $D = \{u_1, u_2, \ldots\ldots, u_s, \ldots, u_k\}$ be a unique γ_r set of G.

Let $\eta = \min_{1\le i \le k} \{|pn(u_i, D)|\}$, then there exist an s, $1 \le s \le k$ such that $pn(u_s, D) = \eta$.

Let $pn(u_s, D)$ set is $\{u_{s1}, u_{s2}, u_{s3}, \ldots, u_{st}\}$ $t \ge 4$, Consider the set,

$(D - \{u_s\}) = \{u_1, u_2, \ldots\ldots u_{s-1}, u_{s+1}, \ldots, \ldots, u_k\}$ and its cardinality is $k - 1$, where $(D - \{u_s\})$ set dominates all its private neighbor and the induced subgraph of the private neighbour is either a path or union of k_2. Consider the set $(pn(u_s, D) - \{u_{sj}\})$, $1 \le j \le t$. Then consider the set $(D - \{u_s\}) \cup (pn(u_s, D) - \{u_{sj}\})$ that is,

$D_1 = \{u_1, u_2, \ldots\ldots u_{s-1}, u_{s+1}, \ldots, \ldots, u_k, u_{s1}, u_{s2}, u_{s3}, \ldots, u_{sj-1}, u_{sj+1} \ldots, u_{st}\}$ is a new restrained dominating set. Therefore, the cardinality of the set D_1 is the sum of the cardinality of the $(D - \{u_s\})$ set and the cardinality of $(pn(u_s, D) - \{u_{sj}\})$.

Hence $|D_1| = k + t - 2 = k + \eta - 2$. Moreover, $k + \eta - 2$ is always greater than or equal to $k + 2$, since $\eta \ge 4$.

Proposition 3.8

Let G be a n - fan graph with nodes $u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_k$, then $\gamma_r(\langle pn(u_i, D)\rangle \ge 2$.

Proof

Let G be a n - fan graph with nodes. Let D be the set of nodes. Therefore, $D = \{u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_k\}$. D is a unique γ_r set of G. Since for any $u_i \in D$, the $\langle pn(u_i, D)\rangle$ is either a path of length greater than or equal to 4 or union of k_2, then the γ_r set is always greater than or equal to 4.

Proposition 3.9

Let G be a n - fan graph with unique γ_r set, $D = \{u_i\}_{i=1}^{m}$, $m \ge 2$ and $\langle pn(u_i, D)\rangle$ be a path with s_i vertices, where $s_i \ne s_j$, for $i \ne j$, $1 \le i, j \le m$, $s_i, s_j \ge 4$ and atleast one $s_i = 4$.

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Then G is γ_r - γ_r enresdowed but not m - γ_r enresdowed for any m, $\gamma_r+1 \leq m \leq \sum_{\substack{i=1 \\ i\neq m}}^{m} s_i - (\gamma_r + 1)$.

Proof

Let G be a graph with unique minimum restrained dominating set, $D = \{u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_m\}$,$m \geq 2$. Let $\langle pn(u_i, D)\rangle$ is a path with s_i vertices , where $s_i \neq s_j$, for $i \neq j$, $1\leq i, j \leq k$, $s_i, s_j \geq 4$.

For the vertex u_1 ,let $\langle pn(u_1, D)\rangle$ be a path with the vertex $\{v_1, v_2, \ldots\ldots, v_{s_1}\}$, $s_1 \geq 4$. In general for the vertex u_i , let $\langle pn(u_i, D)\rangle$ be a path with the vertex , $\{v_{s_{i-1}+1}, v_{s_{i-1}+2}, \ldots\ldots, v_{s_i}\}, s_i \geq 4$. The vertex set of G is $\{u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_m, v_1, v_2, \ldots\ldots, v_{s_1}, \ldots, v_{s_{i-1}+1}, v_{s_{i-1}+2}, \ldots, v_{s_i}, \ldots\ldots, v_{s_{m-1}+1}, v_{s_{m-1}+2}, \ldots\ldots, v_{s_m}\}$, where each s_i vertices are dominated by u_i, $1\leq i \leq m$. Since $D = \{u_1, u_2, \ldots\ldots, u_i, \ldots\ldots, u_m\}$ is the γ_r set of G .Clearly G is γ_r - γ_r enresdowed.

Without loss of generality,let $s_i = 4$. Consider a set D_1 in such a manner where $D_1 = \{u_1, u_2, \ldots, u_{i-1}, u_{i+1} \ldots, u_m, v_{s_{i-1}+1}, v_{s_i}\}$of cardinality m_1,where $m_1 = \gamma_r+1$ and $v_{s_{i-1}+1}, v_{s_i}$ belong to a set of vertices of $\langle pn(u_i, D)\rangle$. If suppose $\langle pn(u_i, D)\rangle$ is a P_4, then $D_1$is a restrained dominating set without D of cardinality γ_r+1. Therefore G is not m_1 - γ_r enresdowed for any m_1 , $m_1 = \gamma_r+1$.

Consider a sets of cardinality m , where $m > \gamma_r+1$, the path $\langle pn(u_i, D)\rangle$ $1\leq i \leq m - 1$ is of three types namely , P_{3i}, $(i \geq 2)$, P_{3i+1}, and P_{3i+2}, $(i \geq 1)$. Consider the path P_{3i}, $(i \geq 2)$. Let the vertex set of P_{3i}, $(i \geq 2)$ be $P_{3i} = \{v_{s_i-1}+1, v_{s_i-1}+2, \ldots, v_{s_i}\}, s_i \geq 4$. For the path of type P_{3i} consider a set $D_{3i} = \{u_1, u_2, \ldots, u_{3i-1}, u_{3i+1}, \ldots, u_{m-1}\} \cup \{v_{s_{3i}-1}+2, v_{s_{3i}-1}+3, \ldots, v_{s_{3i}-1}\}$.$D_{3i}$ is of cardinality $(s_{3i} - 2 + m - 2)$. Consider the path P_{3i+1}, $(i \geq 1)$. For the path of type P_{3i+1} consider a set $D_{3i+1} = \{u_1, u_2, \ldots, u_{3i}, u_{3i+2}, \ldots, u_{m-1}\} \cup \{v_{s_{3i+1}-1}+2, v_{s_{3i+1}-1}+3, \ldots, v_{s_{3i+1}-1}\}$ D_{3i+1} is of cardinality $(s_{3i+1} - 2 + m - 2)$.Similarly consider the path P_{3i+2}, $(i \geq 1)$. For the path of type P_{3i+2} consider a set $D_{3i+2} = \{u_1, u_2, \ldots, u_{3i+1}, u_{3i+3}, \ldots, u_{m-1}\} \cup \{v_{s_{3i+2}-1}+2, v_{s_{3i+2}-1}+3, \ldots, v_{s_{3i+2}-1}\}$. D_{3i+2} is of cardinality $(s_{3i+2} - 2 + m - 2)$.Let $D_m = \{u_1, u_2, \ldots\ldots, u_{m-1}\} \cup \{v_{s_m-1}+2, v_{s_m-1}+3, \ldots, v_{s_m-1}\}$.$D_m$ is of cardinality $(s_m - 2 + m - 1)$.

Let $D_1 = \left(\bigcup_{\substack{i>1 \\ 3i\neq m}} D_{3i}\right) \cup D_m$, $D_2 = \left(\bigcup_{\substack{i\geq 1 \\ 3i+1\neq m}} D_{3i+1}\right) \cup D_m$, $D_3 = \left(\bigcup_{\substack{i\geq 1 \\ 3i+2\neq m}} D_{3i+2}\right) \cup D_m$.

Take $D_4 = \left(\bigcup_{\substack{i>1 \\ 3i\neq m}} D_{3i}\right) \cup \left(\bigcup_{\substack{i\geq 1 \\ 3i+1\neq m}} D_{3i+1}\right) \cup \left(\bigcup_{\substack{i\geq 1 \\ 3i+2\neq m}} D_{3i+2}\right) \cup D_k$. D_4 is a restrained dominating set of cardinality $\sum_{\substack{i=1 \\ i\neq m}}^{m} s_i - (\gamma_r + 1)$, which does not contain the u_m. Hence D_4 is a restrained dominating set not containing D. Hence G is not m - γ_r enresdowed for the values of m , $\gamma_r+1 \leq m \leq \sum_{\substack{i=1 \\ i\neq m}}^{m} s_i - (\gamma_r + 1)$.

4. Conclusion

In this paper, we discovered a few outcomes for the k - γ_r enresdowed graph related to private neighborhood.

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