

## Mod(k) Vertex Magic Labeling in Generalized 2-complement of some Graphs- Paper II

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### ABSTRACT

A  $(p,q)$  graph  $G$  with the  $p$  vertices and  $q$  edges is Mod(k) vertex magic for any integer  $k \geq 2, l \in \mathbb{Z}_k$  and there exists a injective map  $f$  from  $V(G)$  to  $\{\lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l, \lfloor \frac{k}{2} \rfloor + l + 1, \dots, \lfloor \frac{k}{2} \rfloor + k(p-1)\}$  such that for any edge  $e$ , and the sum of the labels of vertices adjacent with the  $e$  are all equal to the same constant modulo  $k$ . In this paper, we prove that Generalized 2-complement some graphs namely  $S(K_{1,n})$ ,  $Spl(C_n)$  are Mod(k) vertex magic graphs.

**Keywords:** Graph labeling, Mod(k) vertex magic labeling, star, Subdivision, Splitting graph.

**AMS Subject Classification:** 05C78

### 1.INTRODUCTION

In this paper, we manage just, connected and non-trivial graph  $G=(V(G),E(G))$  among vertex set  $V(G)$  and edge set  $E(G)$ .

Let  $G=(V,E)$  to be a graph and  $P=(W_1,W_2,W_3\dots W_k)$  be partition of  $V$  of order  $k > 1$ . The  $k$ -complement  $G_kP$  of  $G$  (concerning  $P$ ) is characterized as takes after: For all  $W_i$  and  $W_j$  in  $P$ ,  $i \neq j$  expel the edges amongst  $W_i$  and  $W_j$  in  $G$  and join the edges amongst  $W_i$  and  $W_j$  which are not in  $G$ . The graph accordingly acquired is known as the  $k$ - complement of  $G$  regarding  $P$  [2].

A labeling of a graph  $G$  is a mapping that takes an set of graph components for the most part vertices and edges into an set of numbers, generally integers. Numerous sorts of labeling have been examined and a splendid overview of graph labeling is built up [4].

The idea of graph labeling has fulfilled a considerable measure of ubiquity in the area of graph theory. This graph labeling are extremely valuable in Mathematical models for an extensive variety of uses being X-ray, Crystallography, Coding theory, Cryptography, Communication networks design, Radar, Space science, Circuit design and, Database Administration.

In 1970, Kotzig and Rosa defined a magic labeling of a graph  $G(V,E)$  as abijection,  $f:V \cup E \rightarrow \{1,2,3\dots p+q\}$  such that for all edges  $uw$ ,  $f(u)+f(w)+f(uw)$  are the equal [1].

Lee, Su, Wang in 2010 defined  $(p,q)$  graph  $G$  is called Mod(k) edge magic (in short Mod(k)-EM) if here was an edge labeling  $l:E(G) \rightarrow \{1,2,3\dots q\}$  such that for any vertex  $u$ , sum of the

labels of their edges incident with the  $u$  are all equal to the same constant modulo  $k$ . (i.e)  $l^+(u)=c$  for some fixed  $c$  in  $Z_k$  [8].

In 2015, Lau, Alikhani, Lee, Kocay characterized a  $(p,q)$  graph to be  $k$ -edge magic if for any integer  $k \geq 0$ , define a one-to-one map guided from  $E(G)$  to  $\{k, k+1, \dots, k+q-1\}$  and characterized the vertex total for a vertex  $v$  as the total of the labels of the edges incident to  $v$ . In the event that such an edge labeling makes a vertex labeling in which every vertex has a constant vertex total  $(\text{mod } p)$  [2].

In 2016, P.Sumathi and B.Fathima [5] characterized Mod( $k$ ) Vertex magic labeling. A  $(p,q)$  graph  $G=(V,E)$  is said to be a mod( $k$ ) vertex magic if for any integer  $k \geq 2, l \in Z_k$  and there exists an one-to-one map  $f:V(G) \rightarrow \{\lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l, \lfloor \frac{k}{2} \rfloor + l + 1, \lfloor \frac{k}{2} \rfloor + k, \lfloor \frac{k}{2} \rfloor + k + l, \lfloor \frac{k}{2} \rfloor + k + l + 1, \dots, \lfloor \frac{k}{2} \rfloor + k(p-1)\}$  to such an extent that the induced mapping  $f^*:E(G) \rightarrow Z_k$  characterized by  $f^*(uv)=(f(u)+f(v))(\text{mod } k)=l$  is a constant mapping. The function  $f$  is known as a mod( $k$ ) vertex magic labeling (in short Mod( $k$ ) VML) of  $G$ .

In this paper, we contemplate mod( $k$ ) vertex magic labeling of 2-complement of a few graphs to be specific  $S(K_{1,n}), Spl(C_n)$ .

## 2. PRELIMINARIES

In this section, we give the essential definitions and notations identified with this paper.

**Definition 2.1.** From the graph  $G$ , a new graph were obtained by subdividing any edge  $G$  with a new vertex is called Subdivision of the  $G$  and it were denoted by  $S(G)$  [4].

**Definition 2.2.** For a graph  $G$ , the Splitting graph of  $G$  ( $Spl(G)$ ) were attained from the  $G$  joining of any vertex  $u$  of  $G$  is the new vertex of  $u'$  is adjacent to every vertex and is adjacent to  $u$  [4].

## 3. MAIN RESULTS

In this section, we given the existence of Mod( $k$ ) vertex magic labeling of 2-complement of some graphs.

**Theorem: 3.1.** Let  $G$  be a Subdivision of a star ( $S(K_{1,n})$ ) with  $(2n+1, n \geq 1)$  vertices say  $\{u, u_1, u_2, u_3, \dots, u_n, v_1, v_2, v_3, \dots, v_n\}$ . If  $W_1 = \{u\}$  and  $W_2 = \{u_i, v_i : 1 \leq i \leq n\}$  be the partition of  $G_2^P(S(K_{1,n}))$  then 2-complement( $G_2^P$ ) of  $S(K_{1,n})$  admits Mod( $k$ ) vertex magic labeling.

**Proof:** Let  $K_{1,n}$  be a star with  $\{u\} \cup \{u_i, 1 \leq i \leq n\}$  be the vertices and  $\{uu_i, 1 \leq i \leq n\}$  be the edges.

Let  $G=S(K_{1,n})$  be the Subdivision of  $K_{1,n}$  is attained by subdividing any edge of  $K_{1,n}$  with a new vertex  $\{v_i : 1 \leq i \leq n\}$  where  $V(G)=\{u\} \cup \{u_i, 1 \leq i \leq n\} \cup \{v_i, 1 \leq i \leq n\}$  and  $E(G)=\{uu_i, 1 \leq i \leq n\} \cup \{u_i v_i, 1 \leq i \leq n\}$ . It has  $2n+1$  vertices and  $2n$  edges. Let

$G_1 = (V(G_1), E(G_1))$  be the 2-complement ( $G_2^P$ ) of  $S(K_{1,n})$  has two partitions  $W_1 = \{u\}$  and  $W_2 = \{u_i, v_i, 1 \leq i \leq n\}$  where  $V(G_1) = \{u\} \cup \{u_i, v_i, 1 \leq i \leq n\}$  and  $E(G_1) = \{uv_i, 1 \leq i \leq n\}$ .

**Case(i):** When  $k$  is odd.

Define  $f: V(G_1) \rightarrow \{\lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l + 1, \lfloor \frac{k}{2} \rfloor + k, \lfloor \frac{k}{2} \rfloor + k + l + 1, \dots, \lfloor \frac{k}{2} \rfloor + k(2n)\}$  by

$$f(w) = \begin{cases} \lfloor \frac{k}{2} \rfloor, & \text{if } w = u, \text{ for } 0 \leq l \leq k - 1, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2} (2i), & \text{if } w = u_i \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + k (i), & \text{if } w = u_i \text{ for } l = k - 1, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2} (2i - 2) + l + 1, & \text{if } w = v_i \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + k (n + i), & \text{if } w = v_i \text{ for } l = k - 1, 1 \leq i \leq n. \end{cases}$$

Clearly the mapping  $f$  is an injective and we get,

$$f(u)+f(v) = \begin{cases} k(i) + l, & \text{if } u = v, v = v_i \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\ k(n + i) + k - 1, & \text{if } u = v, v = v_i \text{ for } l = k - 1, 1 \leq i \leq n. \end{cases}$$

By the definition of  $\text{Mod}(k)$  vertex magic labeling, the induced mapping  $f^*$  is a constant mapping.

Thus  $f$  is a  $\text{Mod}(k)$  vertex magic labeling.

2-complement ( $G_2^P$ ) of  $S(K_{1,n})$  is  $\text{Mod}(k)$  vertex magic graph if  $k$  is odd.

**Case(ii):** When  $k$  is even.

Define  $f: V(G_1) \rightarrow \{\lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l, \lfloor \frac{k}{2} \rfloor + k, \lfloor \frac{k}{2} \rfloor + k + l, \dots, \lfloor \frac{k}{2} \rfloor + k(2n)\}$  by

$$f(w) = \begin{cases} \lfloor \frac{k}{2} \rfloor, & \text{if } w = u, \text{ for } 0 \leq l \leq k - 1, \\ \lfloor \frac{k}{2} \rfloor + k (i), & \text{if } w = u_i \text{ for } l = 0, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2} (2i), & \text{if } w = u_i \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + k (n + i), & \text{if } w = v_i \text{ for } l = 0, 1 \leq i \leq n \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2} (2i - 2) + l, & \text{if } w = v_i \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n. \end{cases}$$

Obviously the mapping  $f$  is an injective and we get,

$$f(u)+f(v) = \begin{cases} k(n + i + 1), & \text{if } u = v, v = v_i \text{ for } l = 0, 1 \leq i \leq n \\ k(i) + l, & \text{if } u = v, v = v_i \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n. \end{cases}$$

Since the definition of mod(k) vertex magic labeling, the induced mapping  $f^*$  is a constant mapping.

Under the mapping  $f$ , there exists Mod(k) vertex magic labeling for  $(G_2^P)$  of  $S(K_{1,n})$ . Thus 2-complement  $(G_2^P)$  of  $S(K_{1,n})$  is Mod(k) vertex magic graph if  $k$  is even.

Hence 2-complement  $(G_2^P)$  of  $S(K_{1,n})$  admits Mod(k) vertex magic labeling.

**Illustration: 1.** The following figures show the  $S(K_{1,3})$  and 2-complement of  $S(K_{1,3})$  is a Mod(5) vertex magic for  $l=2$ .

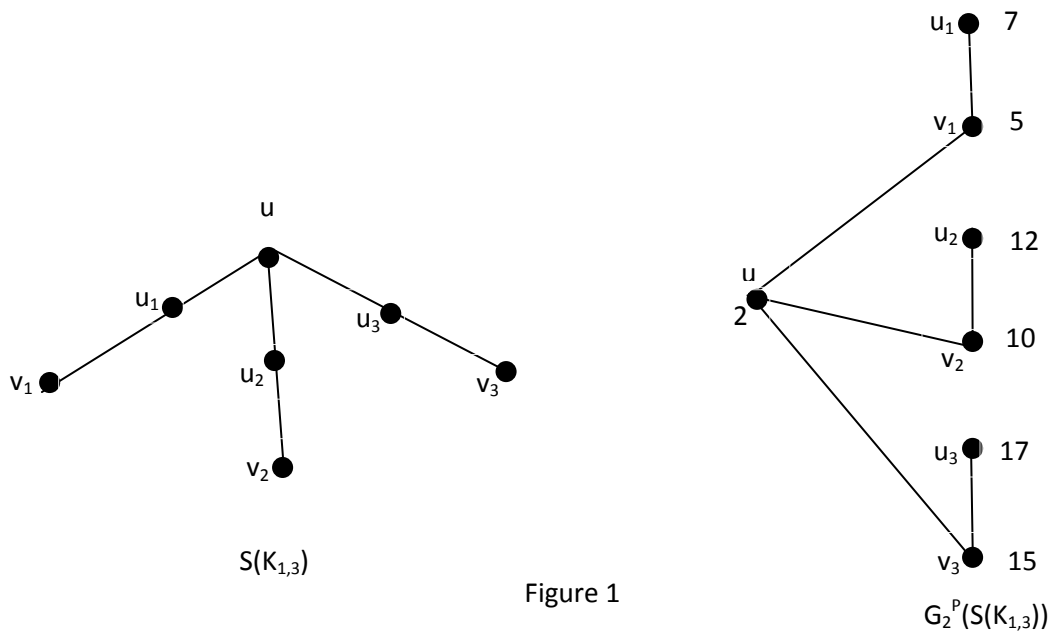


Figure 1

**Theorem: 3.2**

Let  $G$  be a  $Spl(C_n)$  graph with  $\{u_1, u_2, u_3 \dots u_n\}$  and  $\{v_1, v_2, v_3 \dots v_n\}$  is the vertices and  $n$  even for all  $n \geq 6$ . If  $W_1 = \{u_i, v_i; i \text{ is odd}\}$  and  $W_2 = \{u_i, v_i; i \text{ is even}\}$  be the partition of  $G_2^P(Spl(C_n))$  then 2-complement  $G_2^P$  of  $Spl(C_n)$  admits Mod(k) vertex magic labeling.

**Proof:** Let  $(C_n)$  be the cycle of length  $n$  and  $n$  is even for all  $n \geq 6$ . Let  $\{u_i; 1 \leq i \leq n\}$  be the vertices and  $\{u_i, u_{i+1}; 1 \leq i \leq n\} \cup \{u_n, u_1\}$  be the edges of  $C_n$ .

Let  $G = Spl(C_n) = (V(G), E(G))$  is obtained by adding each vertex  $\{u_i; 1 \leq i \leq n\}$  of  $C_n$  a new vertex  $\{v_i; 1 \leq i \leq n\}$  adjacent to each vertex that is adjacent to the  $\{u_i; 1 \leq i \leq n\}$  where  $V(G) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\}$  and  $E(G) = \{u_i, u_{i+1}; 1 \leq i \leq n - 1\} \cup \{v_i, u_{i+1}; 1 \leq i \leq n - 1\} \cup \{v_{i+1}, u_i; 1 \leq i \leq n - 1\} \cup \{u_n, u_1, v_n, u_1, v_1, u_n\}$ . It has  $2n$  vertices and  $3n$  edges.

Let  $G_1=(V(G_1),E(G_1))$  be the 2-complement  $(G_2^P)$  of Spl  $(C_n)$  has two partitions

$W_1=\{u_1, u_3, u_5, \dots, u_{n-1}, v_1, v_3, v_5, \dots, v_{n-1}\}$  and  $W_2=\{u_2, u_4, u_6, \dots, u_n, v_2, v_4, v_6, \dots, v_n\}$  where

$V(G_1)=\{u_i: 1 \leq i \leq n\} \cup \{v_i: 1 \leq i \leq n\}$  and  $E(G_1) = E_1(G_1) \cup E_2(G_1) \cup E_3(G_1) \cup E_4(G_1) \cup$

$E_5(G_1) \cup E_6(G_1) \cup E_7(G_1) \cup E_8(G_1) \cup E_9(G_1) \cup E_{10}(G_1)$  where

$$E_1(G_1)=\{u_1 u_{2i+2} : 1 \leq i \leq \frac{n-4}{2}\}, E_2(G_1)=\{u_i u_{i-(2j+1)} : i=5,7,9, \dots, n-1, 1 \leq j \leq \frac{i-3}{2}\},$$

$$E_3(G_1)=\{u_i u_{i+2j+1} : i=3,5,7, \dots, n-3, 1 \leq j \leq \frac{n-(i+1)}{2}\}, E_4(G_1)=\{u_1 v_{2i+2} : 1 \leq i \leq \frac{n-4}{2}\},$$

$$E_5(G_1)=\{u_i v_{i-(2j+1)} : i=5,7,9, \dots, n-1, 1 \leq j \leq \frac{i-3}{2}\},$$

$$E_6(G_1)=\{u_i v_{i+2j+1} : i=3,5,7, \dots, n-3, 1 \leq j \leq \frac{n-(i+1)}{2}\},$$

$$E_7(G_1)=\{v_1 u_{2i+2} : 1 \leq i \leq \frac{n-4}{2}\}, E_8(G_1)=\{v_i u_{i-(2j+1)} : i=5,7,9, \dots, n-1, 1 \leq j \leq \frac{i-3}{2}\},$$

$$E_9(G_1)=\{v_i u_{i+2j+1} : i=3,5,7, \dots, n-3, 1 \leq j \leq \frac{n-(i+1)}{2}\}, E_{10}(G_1)=\{v_i v_{2i} : i \text{ is odd}\}.$$

**Case(i):** When  $k$  is odd.

Define  $f: V(G_1) \rightarrow \{\lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l + 1, \lfloor \frac{k}{2} \rfloor + k, \lfloor \frac{k}{2} \rfloor + k + l + 1, \dots, \lfloor \frac{k}{2} \rfloor + k(2n-1)\}$  by

$$f(w)=\begin{cases} \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(i-1), & \text{if } w = u_i \text{ for } 0 \leq l \leq k-2, i \text{ is odd,} \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(i-2) + l + 1, & \text{if } w = u_i \text{ for } 0 \leq l \leq k-2, i \text{ is even,} \\ \lfloor \frac{k}{2} \rfloor + k(i-1), & \text{if } w = u_i \text{ for } l = k-1, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(n+i-1), & \text{if } w = v_i \text{ for } 0 \leq l \leq k-2, i \text{ is odd,} \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(n+i-2) + l + 1, & \text{if } w = v_i \text{ for } 0 \leq l \leq k-2, i \text{ is even,} \\ \lfloor \frac{k}{2} \rfloor + k(n+i-1), & \text{if } w = v_i \text{ for } l = k-1, 1 \leq i \leq n. \end{cases}$$

Hence  $f$  is an one - to - one map and we get,

$f(u)+f(v)=$

$$\begin{cases} k(i+2) + l \text{ if } u = u_1, v = u_{2i+2} \text{ for } 0 \leq l \leq k-2, 1 \leq j \leq \frac{n-4}{2}, \\ k(2i+1) + k-1, \text{ if } u = u_1, v = u_{2i+2} \text{ for } l = k-1, 1 \leq j \leq \frac{n-4}{2}, \\ k(i-j-1) + l, \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } 0 \leq l \leq k-2, i = 5,7,9 \dots, n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(2i-2j-3) + k-1, \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } l = k-1, i = 5,7,9 \dots, n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(i+j+2) + l, \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } 0 \leq l \leq k-2, i = 3,5,7, \dots, n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(2i+2j-1) + k-1, \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } l = k-1, i = 3,5,7, \dots, n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ \frac{k}{2}(n+2i+2) + l \text{ if } u = u_1, v = v_{2i+2} \text{ for } 0 \leq l \leq k-2, 1 \leq j \leq \frac{n-4}{2}, \\ k(n+2i+1) + k-1 \text{ if } u = u_1, v = v_{2i+2} \text{ for } l = k-1, 1 \leq j \leq \frac{n-4}{2}. \end{cases}$$

$f(u)+f(v)=$

$$\left\{ \begin{array}{l} \frac{k}{2}(n+2i-2j-2)+l, \text{ if } u=u_i, v=v_{i-(2j+1)} \text{ for } 0 \leq l \leq k-2, i=5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i-2j-3)+k-1, \text{ if } u=u_i, v=v_{i-(2j+1)} \text{ for } l=k-1, i=5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i+2j)+l, \text{ if } u=u_i, v=v_{i+2j+1} \text{ for } 0 \leq l \leq k-2, i=3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2j-1)+k-1, \text{ if } u=u_i, v=v_{i+2j+1} \text{ for } l=k-1, i=3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ \frac{k}{2}(n+2i+2)+l \text{ if } u=v_1, v=u_{2i+2} \text{ for } 0 \leq l \leq k-2, 1 \leq j \leq \frac{n-4}{2}, \\ k(n+2i+1)+k-1, \text{ if } u=v_1, v=u_{2i+2} \text{ for } l=k-1, 1 \leq j \leq \frac{n-4}{2}, \\ \frac{k}{2}(n+2i-2j-2)+l, \text{ if } u=v_i, v=u_{i-(2j+1)} \text{ for } 0 \leq l \leq k-2, i=5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i-2j-3)+k-1, \text{ if } u=v_i, v=u_{i-(2j+1)} \text{ for } l=k-1, i=5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i+2j)+l, \text{ if } u=v_i, v=u_{i+2j+1} \text{ for } 0 \leq l \leq k-2, i=3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2j+1)+k-1, \text{ if } u=v_i, v=u_{i+2j+1} \text{ for } l=k-1, i=3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ \frac{k}{2}(2n+3i-1), \text{ if } u=v_i, v=v_{2i} \text{ for } 0 \leq l \leq k-2, i \text{ is odd}, \\ k(2n+3i-1), \text{ if } u=v_i, v=v_{2i} \text{ for } l=k-1, i \text{ is odd}. \end{array} \right.$$

Since the definition of Mod(k) vertex magic labeling, the induced mapping  $f^*$  is a constant mapping.

Under the mapping  $f$ , there exists mod(k) vertex magic labeling for 2-complement of  $Spl(C_n)$ .

Thus  $G_2^P$  of  $Spl(C_n)$  is Mod(k) vertex magic graph if  $k$  is odd.

**Case(ii):** When  $k$  is even.

Define  $f: V(G) \rightarrow \{ \lfloor \frac{k}{2} \rfloor, \lfloor \frac{k}{2} \rfloor + l, \lfloor \frac{k}{2} \rfloor + k, \lfloor \frac{k}{2} \rfloor + k + l, \dots, \lfloor \frac{k}{2} \rfloor + k(2n-1) \}$  by

$$f(w) = \left\{ \begin{array}{l} \lfloor \frac{k}{2} \rfloor + k(i-1), \text{ if } w = u_i \text{ for } l=0, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(i-1), \text{ if } w = u_i \text{ for } 1 \leq l \leq k-1, i \text{ is odd}, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(i-2) + l, \text{ if } w = u_i \text{ for } 1 \leq l \leq k-1, i \text{ is even}, \\ \lfloor \frac{k}{2} \rfloor + k(n+i-1), \text{ if } w = v_i \text{ for } l=0, 1 \leq i \leq n, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(n+i-1), \text{ if } w = v_i \text{ for } 1 \leq l \leq k-1, i \text{ is odd}, \\ \lfloor \frac{k}{2} \rfloor + \frac{k}{2}(n+i-2) + l, \text{ if } w = v_i \text{ for } 1 \leq l \leq k-1, i \text{ is even}. \end{array} \right.$$

Clearly  $f$  is an injective mapping and we get,

$f(u)+f(v)=$

$$\left\{ \begin{array}{l} k(2i+2), \text{ if } u = u_1, v = u_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}, \\ k(i+2) + l \text{ if } u = u_1, v = u_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\ k(2i-2j-2), \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(i-j-1) + l, \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(2i+2j), \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(i+j+2) + l, \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2) \text{ if } u = u_1, v = v_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}. \\ \frac{k}{2}(n+2i+2) + l \text{ if } u = u_1, v = v_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\ k(n+2i-2j-2), \text{ if } u = u_i, v = v_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ \frac{k}{2}(n+2i-2j-2) + l, \text{ if } u = u_i, v = v_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i+2j), \text{ if } u = u_i, v = v_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2j) + l, \text{ if } u = u_i, v = v_{i+2j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2), \text{ if } u = v_1, v = u_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}, \\ \frac{k}{2}(n+2i+2) + l \text{ if } u = v_1, v = u_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\ k(n+2i-2j-2), \text{ if } u = v_i, v = u_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ \frac{k}{2}(n+2i-2j-2) + l, \text{ if } u = v_i, v = u_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \dots n-1, 1 \leq j \leq \frac{i-3}{2}, \\ k(n+2i+2j+2), \text{ if } u = v_i, v = u_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(n+2i+2j) + l, \text{ if } u = v_i, v = u_{i+2j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \dots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\ k(2n+3i), \text{ if } u = v_i, v = v_{2i} \text{ for } l = 0, i \text{ is odd}, \\ \frac{k}{2}(2n+3i-1), \text{ if } u = v_i, v = v_{2i} \text{ for } 1 \leq l \leq k-1, i \text{ is odd}. \end{array} \right.$$

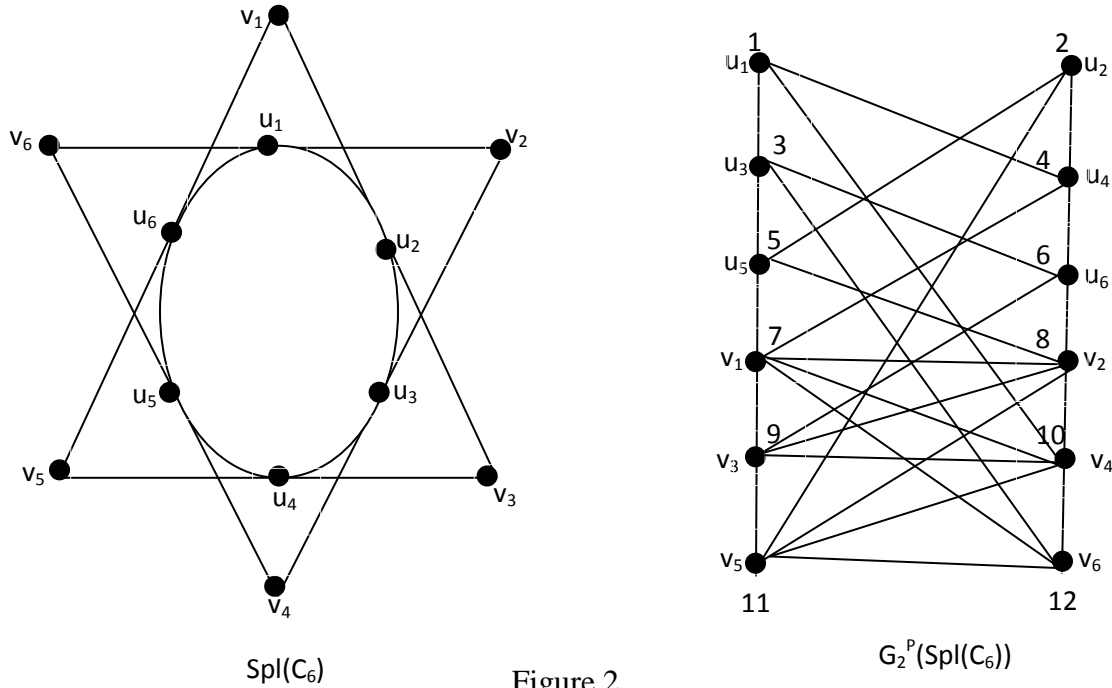
By the definition of  $\text{Mod}(k)$  vertex magic labeling, the induced mapping  $f^*$  is a constant mapping.

Thus  $f$  is a  $\text{mod}(k)$  vertex magic labeling.

$G_2^P$  of  $\text{Spl}(C_n)$  is  $\text{Mod}(k)$  vertex magic graph if  $k$  is even.

Hence 2-complement  $G_2^P$  of  $\text{Spl}(C_n)$  admits  $\text{Mod}(k)$  vertex magic labeling for all  $n \geq 6$ .

**Illustration: 2.** The following figures show  $Spl(C_8)$  and its 2-complement is a Mod(2) vertex magic graph for  $l=1$ .



#### 4. CONCLUSION

In this paper we have discussed that Generalized 2-complement of some graphs are Mod(k) vertex magic graphs. Analogues work can be carried by us for other families also.

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